3.2 Sets of Sets

Example $A = \{ \{1,2\}, \emptyset, \{1,2,3\}, \{1\} \}$

- A has 4 elements
- $1 \notin A$
- $\{1\} \in A$
- $\{1\} \not\subseteq A$ since $1 \notin A$

- A power set of A is denoted $P(A)$, and is a set of all subsets of A.

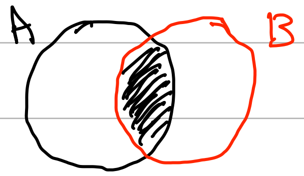
- The $\emptyset$ set is a subset of all sets.

- Let A be a finite set of cardality n. Then the cardality of the power set is 2^n.
  - $|P(A)| = 2^n$

- $A = \{2, 3, 5, 7, 14\}$
  - $2 \in P(A)$ FALSE
  - $\{2\} \in P(A)$ TRUE

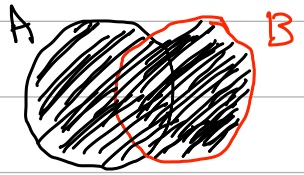
3.3 Union and Intersection

- Let A and B be sets
  - The intersection of A and B, denoted $A \cap B$, and read "A intersect B" is the set of all elements that are elements of both A and B.



$A \cap B$

  - The union of A and B, denoted $A \cup B$ and read "A union B" is the set of all elements that are elements of A or B.



$A \cup B$

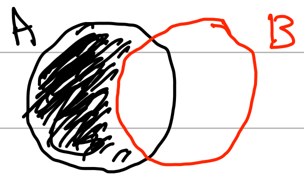
- Special Notation

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \cdots \cap A_n = \{x : x \in A_i, \text{ for all } i \text{ such that } 1 \le i \le n\}$$

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \cdots \cup A_n = \{x : x \in A_i, \text{ for some } i \text{ such that } 1 \le i \le n\}$$

3.4 More Set Operations

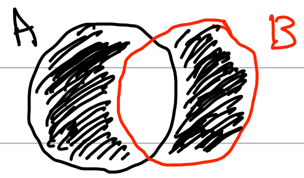
- The <u>difference</u> of two sets A and B, denoted $A - B$, is the set of elements that are in A but not in B.



$A - B$

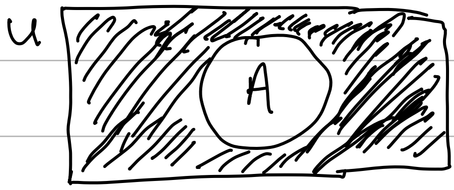
- The <u>symettric difference</u> between two sets A and B, denoted $A \oplus B$, is the set of all elements that are in either A or B, but not both.

$$A \oplus B = (A - B) \cup (B - A)$$



$A \oplus B$

- The <u>Complement</u> of set A, denoted $\bar{A}$, is the set of all elements in U (universal set) that are not elements of A.



3.5 Set Identities

- A **set identity** is an equation involving sets that is true regardless of the contents of the sets in the expression.

Nume	Identities	
Indepotent Laws	$A \cup A = A$	$A \cap A = A$
Associative Laws	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Commutative Laws	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Distributive Laws	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
Identity Laws	$A \cup \varnothing = A$	$A \cap U = A$
Domination Laws	$A \cap \varnothing = \varnothing$	$A \cup U = U$
Double Complement Law	$\overline{\overline{A}} = A$	$\overline{\overline{A}} = A$
Complement Laws	$A \cap \overline{A} = \varnothing$ $\overline{U} = \varnothing$	$A \cup \overline{A} = \varnothing$ $\overline{\varnothing} = U$
DeMorgans Laws	$\overline{A \cup B} = \overline{A} \cap \overline{B}$	$\overline{A \cap B} = \overline{A} \cup \overline{B}$
Absorption Laws	$A \cup (A \cap B) = A$	$A \cap (A \cup B) = A$

3.6 Cartesian products

- A ordered pair is written (x, y)
  - x is the first entry, y is the second entry.

- For two sets, A and B, the cartesian product of A and B, denoted $A \times B$, is the set of all ordered pairs in which the first entry is in A and the second entry is in B.
  - $A \times B = \{(a, b) : a \in A \text{ and } b \in B\}$

- $A = \{1, 2, 3\}$, $B = \{x, y\}$
  - $(1, y) \in A \times B$ TRUE
  - $(1, y) \in B \times A$ FALSE

- A ordered list of 3 items is called an ordered triple, and is denoted (x, y, z)
  - (w, x, y, z) is an ordered 4-tuple
  - (u, w, x, y, z) is an ordered 5-tuple

- $A \times A = A^2$

- $A = \{x, y\}$, the set $A^2 = \{xx, xy, yx, yy\}$
- string: a sequence of characters
- alphabet: the set of characters used in a set of strings
- length: number of characters in a string

- A **binary string** is a string with alphabet $\{0, 1\}$.
- A **bit** is a character in a binary string
- An **n-bit** string is a binary string of length n.
    - 7 bit string : 0010110

- The **empty string** is the unique string with a length of 0, and is usally denoted by the symbol λ.

- If s and t are two strings, then the concatenation of s and t (denoted st) is the string obtained by putting s and t together.